One-mirror Fabry-Perot and one-slit Young interferometry

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We describe a new and distinctive interferometry in which a probe particle scatters off a *superposition* of locations of a *single* free target particle. In one dimension, probe particles incident on superposed locations of a single "mirror" can interfere as if in a Fabry-Perot interferometer; in two dimensions, probe particles scattering off superposed locations of a single "slit" can interfere as if in a two-slit Young interferometer. The condition for interference is *loss* of orthogonality of the target states and reduces, in simple examples, to *transfer* of orthogonality from target to probe states. We analyze experimental parameters and conditions necessary for interference to be observed.

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The two-slit interference experiment contains a mystery of quantum theory, and Feynman even stated that "it contains the *only* mystery" [1]. Whether or not we accept Feynman's statement, we can easily accept the importance of the two-slit experiment and its generalizations to quantum theory. Let us consider a particularly "quantum" generalization of the two-slit experiment: instead of two slits for the interfering quanta, the experiment contains a single free "quantum slit" (or a single Fabry-Perot (FP) mirror) in a superposition of two locations. Can scattering from such a superposition show quantum interference? Cohen-Tannoudji et al. [2] answered this question [3] in the negative, with an assumption that is justified in specific experimental settings. But the advent of new experimental settings, such as one- and two-dimensional potentials on the Atom Chip [4] and highly controlled atom optics, lead us to reconsider the question. Here we derive a general condition for quanta impinging on a superposition of target locations to interfere, and describe the experimental conditions for this distinctive quantum interference to be observed.

To specify the experimental setting, we replace the FP mirrors, or the slits in the Young double-slit experiment, with a single quantum target: a scattering center, i.e. an ultracold atom, in a superposition of orthogonal position states. Both the probe and the target are free. We confine the target to move in one dimension (i.e. in a tight atomic guide). In the first example below, both the probe and target are one-dimensional, and the superposed locations of the target form a one-dimensional, one-mirror FP interferometer. In the second example, the probe moves in a plane containing the target axis and scatters in two dimensions off the superposed locations of the target, which form a double-slit interferometer made of a single slit.

Let the initial target wave function $\varphi_\alpha(\mathbf{X})$ be a superposition of wave packets separated by a distance d,

$$\varphi_\alpha(\mathbf{X}) = \frac{1}{\sqrt{2}}\left[\varphi_L(\mathbf{X}) + e^{i\alpha}\varphi_R(\mathbf{X})\right] \quad , \tag{1}$$

where we take $\varphi_R(\mathbf{X}) = \varphi_L(\mathbf{X}-\mathbf{d}) = e^{\nabla\cdot\mathbf{d}}\varphi_L(\mathbf{X})$ for convenience. The wave packets have support in regions smaller than $d/2$. Such a wave function may be engineered by growing a barrier in the middle of a harmonic oscillator trap, so as to form a double well in the free dimension of the target, and then by quickly shutting off this trap.

How should quantum interference show up in scattering from a superposition of locations? An operational definition is essential. No local measurement on $\varphi_L(\mathbf{X})$ or $\varphi_R(\mathbf{X})$ alone can yield α, the relative phase of the wave packets; no probe particle interacting with a target at *one* of its locations, but not *both*, can provide any information about α. Hence, dependence on α in the final state of the probe is a sure signal of interference between paths of the probe scattering from the two target locations. Since $\varphi_\alpha(\mathbf{X}) = [1 + e^{i\alpha+\nabla\cdot\mathbf{d}}]\varphi_L(\mathbf{X})/\sqrt{2}$, the Fourier transform $\tilde{\varphi}_\alpha(\mathbf{P})$ of $\varphi_\alpha(\mathbf{X})$ is

$$\tilde{\varphi}_\alpha(\mathbf{P}) = [1 + e^{i\alpha+i\mathbf{P}\cdot\mathbf{d}/\hbar}]\tilde{\varphi}_L(\mathbf{P})/\sqrt{2} \quad , \tag{2}$$

and shows peaks in $\mathbf{P}\cdot\mathbf{d}/d$ separated by h/d. A change in α shifts the peaks, i.e. changes the *modular* momentum [5] defined as $\mathbf{P}\cdot\mathbf{d}/d$ modulo h/d. We will see how α can show up in the final momentum distribution of the probe particles.

In the case that a probe and target have initial momenta $\mathbf{p}^{in}$ and $\mathbf{P}^{in}$, respectively, the initial overall state $|\Psi_{in}\rangle$ of the probe and target is

$$|\Psi_{in}\rangle = |\mathbf{p}^{in}\rangle \otimes |\varphi_\alpha\rangle = |\mathbf{p}^{in}\rangle \otimes \int d^3\mathbf{P}^{in}\ \tilde{\varphi}_\alpha(\mathbf{P}^{in})|\mathbf{P}^{in}\rangle \ , \tag{3}$$

In Eq. (3) and below, the first ket in any tensor product refers to the probe and the second ket refers to the target. The state $|\mathbf{p}^{in}\rangle \otimes |\mathbf{P}^{in}\rangle$ can scatter to a state $|\mathbf{p}^{fin}\rangle \otimes |\mathbf{P}^{in} + \mathbf{p}^{in} - \mathbf{p}^{fin}\rangle$. We let $S(\mathbf{p}^{in}, \mathbf{p}^{fin}; \mathbf{P}^{in})$ denote the amplitude of the transition. Then the overall final state $|\Psi_{fin}\rangle$ is

$$\int d^3\mathbf{p}^{fin} \int d^3\mathbf{P}^{in} \quad \tilde{\varphi}_\alpha(\mathbf{P}^{in}) S(\mathbf{p}^{in}, \mathbf{p}^{fin}; \mathbf{P}^{in})\ |\mathbf{p}^{fin}\rangle \otimes |\mathbf{P}^{in} + \mathbf{p}^{in} - \mathbf{p}^{fin}\rangle \quad . \tag{4}$$

Cohen-Tannoudji et al. [2] considered the limit in which $S(\mathbf{p}^{in}, \mathbf{p}^{fin}; \mathbf{P}^{in})$ is independent of $\mathbf{P}^{in}$, a limit appropriate to photons scattering off a heavy atom. With this assumption, the overall final state $|\Psi_{fin}\rangle$ reduces to

$$|\Psi_{fin}\rangle = \int d^3\mathbf{X} \;\; \varphi_\alpha(\mathbf{X}) \;\; |\chi(\mathbf{X})\rangle \otimes |\mathbf{X}\rangle \quad , \tag{5}$$

where $|\chi(\mathbf{X})\rangle$ is a probe (photon) state that depends on the location of the target. They showed that there can be no interference between photon states entangled with the two locations of the target, because the target states remain orthogonal and collapse the superposition. Thus if the scattering matrix does not depend on $\mathbf{P}^{in}$, there can be no interference.

But if the scattering matrix depends on $\mathbf{P}^{in}$, there can be interference in the final momentum distribution of the probe. We now illustrate such interference in a simple one-dimensional model [6]. Scattering in this model is elastic and the scattering matrix is determined—up to an overall coupling constant ϵ—by (nonrelativistic) energy and momentum conservation. Let m and M denote the masses of the probe and target, respectively; apart from their interaction, they are free. The initial state is the one-dimensional version of Eqs. (1-3). The final state is the one-dimensional version of Eq. (4) except that p^{fin} is determined by p^{in} and P^{in}:

$$p^{fin} = \frac{2m}{M+m} P^{in} - \frac{M-m}{M+m} p^{in} \quad . \tag{6}$$

Thus the scattered part of the final state is

$$\epsilon \int dP^{in} \;\; \tilde{\varphi}_\alpha(P^{in}) \;\; |p^{fin}\rangle \otimes |P^{in} + p^{in} - p^{fin}\rangle \quad , \tag{7}$$

and the probability that the probe scatters with a particular momentum p^{fin} is proportional to $|\tilde{\varphi}_\alpha(P_*^{in})|^2$, where P_*^{in} is the value of P^{in} that solves Eq. (6):

$$\begin{aligned} &\mathrm{prob}(p^{fin}) \\ &= \epsilon^2 \frac{M+m}{2m} \left| \tilde{\varphi}_\alpha \left[\frac{M+m}{2m} p^{fin} + \frac{M-m}{2m} p^{in} \right] \right|^2 . \end{aligned} \tag{8}$$

Eq. (8) shows that the momentum distribution of the scattered probe reproduces the momentum distribution of the target, only shifted by $(M-m)p^{in}/2m$ and scaled by $(M+m)/2m$; and from Eq. (2), $|\tilde{\varphi}_\alpha(P)|^2$ equals $[1 + \cos(Pd/\hbar + \alpha)]|\tilde{\varphi}_L(P)|^2$, where $\tilde{\varphi}_L(P)$ is broad compared to $\hbar/d$ because $\varphi_L(X)$ is narrow compared to d. The distribution of p^{fin} depends on α, as claimed.

In any realistic experiment, the incident probe state has a momentum spread $\Delta p^{in} > 0$. To model this spread we fold $\mathrm{prob}(p^{fin})$ in Eq. (8) with a distribution $g(p^{in})$:

$$g(p^{in}) = e^{-(p^{in} - \langle p^{in}\rangle)^2 / 2(\Delta p^{in})^2} \quad . \tag{9}$$

Folding $\mathrm{prob}(p^{fin})$ with $g(p^{in})$ (i.e. summing probabilities rather than amplitudes) is allowed because we trace

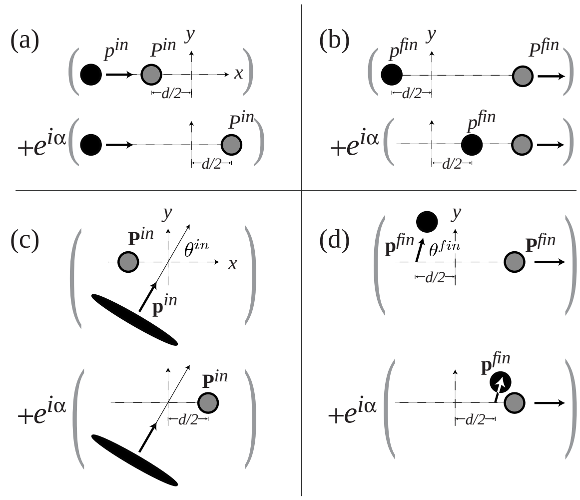

FIG. 1: (a-b) One-dimensional model: (a) A probe (in black) approaches a target (in gray) superposed at locations $X = \pm d/2$ with relative phase α. (b) The probe and target after scattering. (c-d) Two-dimensional model: (c) A probe wave packet, with incident angle θ^{in} and momentum p^{in}, approaches a stationary target superposed at $X = \pm d/2$. (d) If the target scatters with momentum $P^{fin} = Mp^{in}/m \cos\theta^{in}$, the orthogonality of the target states is transferred to the probe.

over the final target state and no two values of p^{in} correspond to the same p^{fin} and P^{fin} (i.e. no two values of p^{in} interfere in the same final state). To estimate the visibility of interference fringes, we can approximate $|\tilde{\varphi}_\alpha(P_*^{in})|^2$ by $1 + \cos(P_*^{in} d/\hbar + \alpha)$ in this convolution. Then the probability of p^{fin} is proportional to

$$1 + A \cos\left(\left[\frac{M+m}{2m} p^{fin} + \frac{M-m}{2m} \langle p^{in} \rangle \right] \frac{d}{\hbar} + \alpha \right) \quad , \tag{10}$$

where $A = e^{-d^2(M-m)^2(\Delta p^{in})^2/8m^2\hbar^2}$. The visibility of the fringes in the distribution of p^{fin} is, by definition, the difference between neighboring maxima and minima divided by their sum, so it equals A. Since the visibility is suppressed exponentially in $d^2(M-m)^2(\Delta p^{in})^2/8m^2\hbar^2$, interference fringes are not visible for $m \ll M$. Indeed, $m \ll M$ and Eq. (6) together imply that the scattering matrix is insensitive to P^{in}, as Cohen-Tannoudji et al. [2] assumed. But for $m = M$ there is no suppression of visibility.

Note that when probe particles of mass m scatter off *two* target particles of mass M, visibility is optimal [7] for $m \ll M$; here visibility vanishes for $m \ll M$. This distinction underscores the novelty of our interference effect.

We can describe the interference effect more generally as a transfer of orthogonality. Initially, the wave function of the target is $|\varphi_\alpha\rangle = [|\varphi_L\rangle + e^{i\alpha}|\varphi_R\rangle]/\sqrt{2}$, with $|\varphi_L\rangle$ and $|\varphi_R\rangle$ orthogonal. If the initial state of the probe is $|\psi_{in}\rangle$, the overall initial state is $|\Psi_{in}\rangle = |\psi_{in}\rangle \otimes [|\varphi_L\rangle + e^{i\alpha}|\varphi_R\rangle]/\sqrt{2}$ and it evolves according to

some unitary operator U until the probe is detected in a final state $|\psi_{fin}\rangle$. The probability to detect this final state is tr $(\rho|\psi_{fin}\rangle\langle\psi_{fin}|)$ where tr indicates the trace over the probe and target Hilbert spaces and

$$\rho = U|\psi_{in}\rangle \otimes [|\varphi_L\rangle + e^{i\alpha}|\varphi_R\rangle][\langle\varphi_L| + e^{-i\alpha}\langle\varphi_R|] \otimes \langle\psi_{in}|U^\dagger \tag{11}$$

is a density matrix. Now consider $\mathrm{tr}_\varphi(U|\psi_{in}\rangle \otimes |\varphi_L\rangle \langle\varphi_R| \otimes \langle\psi_{in}|U^\dagger)$, where tr_φ indicates the trace over only the target Hilbert space. If this latter trace vanishes, then the probability of any final state $|\psi_{fin}\rangle$ of the probe cannot depend on α and there is no interference. But $\mathrm{tr}_\varphi(|\psi_{in}\rangle \otimes |\varphi_L\rangle\langle\varphi_R| \otimes \langle\psi_{in}|)$ vanishes because $|\varphi_L\rangle$ and $|\varphi_R\rangle$ are orthogonal. For interference, then, the superposed states of the target must lose their orthogonality during the evolution U. The states $U|\psi_{in}\rangle \otimes |\varphi_L\rangle$ and $U|\psi_{in}\rangle \otimes |\varphi_R\rangle$, however, remain orthogonal as U is unitary. Hence U must transfer the orthogonality of the target states to other states. In general, the orthogonal states $U|\psi_{in}\rangle \otimes |\phi_L\rangle$ and $U|\psi_{in}\rangle \otimes |\phi_R\rangle$ are entangled states of the probe and target. But if they are product states, then U transfers orthogonality from the target to the probe. Our one-dimensional model illustrates this transfer. Fig. 1(a) depicts a probe approaching a target prepared in the initial state $|\varphi_\alpha\rangle$ of Eq. (1), and Fig. 1(b) shows the particles after scattering. If $m = M$, the probe and target simply exchange states, as they do in classical mechanics, so that orthogonality is transferred from target to probe. We may regard this exchange as an interferometric analogue of entanglement swapping. If $m \neq M$ the probe and target do not scatter to a product state, but partial transfer of orthogonality from target to probe still accounts for the partial visibility at $m \approx M$.

Our general description sheds light also on scattering processes in which the target is not free, e.g. in a high barrier double well potential. Here, no transfer of orthogonality is possible—$|\varphi_L\rangle$ and $|\varphi_R\rangle$ cannot lose their orthogonality—hence no interference. This explanation complements the one in Schomerus et al. [8] ruling out interference on the basis of energy considerations, when the probe has sufficient energy to excite the antisymmetric state of the target.

In our second example, the probe moves in a plane containing the axis to which the target is confined. Hence it is scattered by a "double-slit interferometer" made of a single slit. The momentum of the probe has two components, p_x and p_y, where the axis of the target defines the x-axis. Energy and the x-component of momentum are conserved, but not the y-component (since the target is constrained). We begin with scattering of momentum states of a probe and target. They scatter at $y = 0,\ x = X$. It is helpful to change variables. First, we rescale the position X of the target to $z \equiv X\sqrt{M/m}$ and correspondingly the momentum (whether P^{in} or P^{fin}) to $p_z \equiv P\sqrt{m/M}$. With this rescaling, an initial wave function with momenta p_x^{in}, p_y^{in}, P^{in} can be written

$$\Psi_{in}(\mathbf{r}, t) = e^{i\mathbf{p}^{in}\cdot\mathbf{r}} e^{-i(p^{in})^2 t/2m\hbar} \quad , \tag{12}$$

where $\mathbf{r} = (x, y, z)$ and $\mathbf{p} = (p_x, p_y, p_z)$. It resembles the wave function of a single free particle scattering on the line defined by $y = 0,\ x = z\sqrt{m/M}$. Next we rotate through $\kappa \equiv \arctan\sqrt{m/M}$ in the xz-plane,

$$\begin{aligned} \bar{x} &= x\cos\kappa - z\sin\kappa \quad , \\ \bar{y} &= y \quad , \\ \bar{z} &= x\sin\kappa + z\cos\kappa \quad , \end{aligned} \tag{13}$$

and correspondingly

$$\begin{aligned} \bar{p}_x &= p_x\cos\kappa - p_z\sin\kappa \quad , \\ \bar{p}_y &= p_y \quad , \\ \bar{p}_z &= p_x\sin\kappa + p_z\cos\kappa \quad , \end{aligned} \tag{14}$$

so that the scattering line coincides with the $\bar{z}$-axis; the initial wave function still has the form of Eq. (12) but $\bar{\mathbf{r}}$, $\bar{\mathbf{p}}^{in}$ replace $\mathbf{r}$, $\mathbf{p}^{in}$. The probe and the target interact at short range, hence the scattering is cylindrically symmetric; if the initial momentum is $\bar{\mathbf{p}}^{in}$ then the probability distribution of $\bar{\mathbf{p}}^{fin}$ is

$$\mathrm{prob}(\bar{\mathbf{p}}^{fin}|\bar{\mathbf{p}}^{in}) = \epsilon^2 \frac{\delta(\bar{p}_z^{fin} - \bar{p}_z^{in})\delta(\bar{p}_\rho^{fin} - \bar{p}_\rho^{in})}{2\pi\bar{p}_\rho^{fin}} \quad , \tag{15}$$

where $\bar{p}_\rho \equiv [\bar{p}_x^2 + \bar{p}_y^2]^{1/2}$. Transforming Eq. (15) back to the original coordinates, we obtain

$$\begin{aligned} &\mathrm{prob}(p_x^{fin}, p_y^{fin}, P^{fin}|p_x^{in}, p_y^{in}, P^{in}) = \\ &\epsilon^2 \frac{\delta(P^{fin} + p_x^{fin} - p_x^{in} - P_*^{in})\delta(P^{in} - P_*^{in})}{2\pi\tan^2\kappa\sin\kappa|p_x^{fin} - p_x^{in}|} \ , \end{aligned} \tag{16}$$

where P_*^{in} is the value of P^{in} obtained by solving the two constraints of energy and momentum conservation:

$$P_*^{in} = \frac{1}{2}\left[p_x^{fin} - p_x^{in} + \frac{M}{m}\frac{(p^{fin})^2 - (p^{in})^2}{p_x^{fin} - p_x^{in}}\right] \quad . \tag{17}$$

Now suppose we prepare the target in the state $|\varphi_\alpha\rangle$ and the probe in a state $|\psi_{in}\rangle$ with fixed θ^{in} and a spread Δp^{in} around p^{in}. We obtain the probability distribution $\mathrm{prob}(p_x^{fin}, p_y^{fin})$ for the scattered probe by evolving the overall state $|\psi_{in}\rangle \otimes |\varphi_\alpha\rangle$ in time, projecting onto a final state $|p_x^{fin}, p_y^{fin}\rangle$ of the probe, and tracing $|\langle p_x^{fin}, p_y^{fin}|U|\psi_{in}\rangle \otimes |\varphi_\alpha\rangle|^2$ over the final momentum state $|P^{fin}\rangle$ of the target. Expanding $|\psi_{in}\rangle$ in momentum space, we note that since Eq. 17 is quadratic in p^{in} there are at most two values of p^{in} consistent with the same set p_x^{fin}, p_y^{fin} and P^{fin}. Hence for Δp^{in} small enough to include only one of the two, we can obtain $\mathrm{prob}(p_x^{fin}, p_y^{fin})$ by summing probabilities, namely folding Eq. (16) with $|\tilde{\varphi}_\alpha(P^{in})|^2$ and integrating over P^{fin} [11]:

$$\mathrm{prob}(p_x^{fin}, p_y^{fin}) = \epsilon^2 \frac{M^{1/2}(M+m)^{1/2}|\tilde{\varphi}_\alpha(P_*^{in})|^2}{2\pi m|p_x^{fin} - p_x^{in}|} \quad , \tag{18}$$

which we fold with Eq. (9). Here, as in the first example, the probe inherits the interference in the initial target wave function $\tilde{\varphi}_\alpha(P^{in})$.

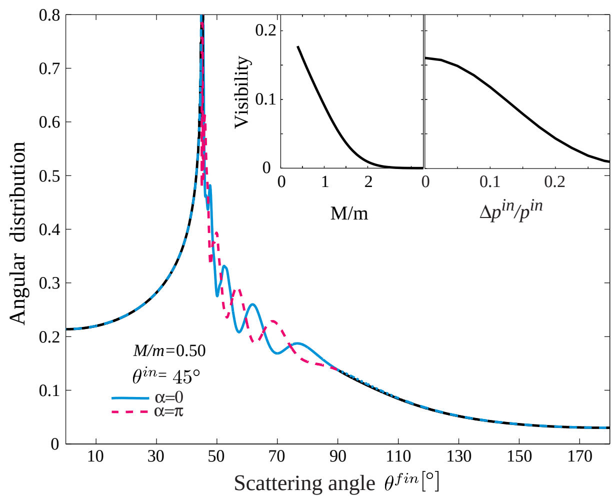

FIG. 2: Normalized angular distribution of scattered probe ($\Delta p^{in} = 0$) incident at $\theta^{in} = 45°$, for relative phases $\alpha = 0$ and $\alpha = \pi$ and $M/m = 0.5$. Insets: Visibility as a function of M/m, and (for $M/m = 0.5$) as a function of $\Delta p^{in}/p^{in}$, at $\theta^{in} = 45°$, $\theta^{fin} = 60°$ [9]. Here we define visibility as $[P_\alpha(\theta^{fin}) - P_{\alpha+\pi}(\theta^{fin})]/[P_\alpha(\theta^{fin}) + P_{\alpha+\pi}(\theta^{fin})]$ maximized over α, where $P_\alpha(\theta^{fin})$ is the probability density for the probe to scatter in the direction θ^{fin} from the initial target state $|\varphi_\alpha\rangle$ of Eq. (1).

If $\tilde{\varphi}_\alpha(P_*^{in})$ changes more rapidly than the denominator of Eq. (18) as a function of p^{in}, we can estimate the visibility of this interference by approximating $|\tilde{\varphi}_\alpha(P_*^{in})|^2$ by $1 + \cos(P_*^{in} d/\hbar + \alpha)$ as before. We obtain $e^{-d^2(\partial P_*^{in}/\partial p^{in})^2(\Delta p^{in})^2/2\hbar^2}$ as the visibility for small Δp^{in}. When $\partial P_*^{in}/\partial p^{in}$ vanishes, the visibility is not suppressed at all, and the spread in the initial state of the probe is transferred to the final state of the target. From $\partial P_*^{in}/\partial p^{in} = 0$ we obtain the condition $P_*^{fin} = Mp^{in}/m\cos\theta^{in}$ (where $P_*^{fin} \equiv P_*^{in} + p_x^{in} - p_x^{fin}$), which we can interpret with the help of Fig. 1. Fig. 1(c) depicts the probe approaching the stationary target at an angle θ^{in}, and Fig. 1(d) depicts the scattering. The target, initially at $X = d/2$ or at $X = -d/2$, scatters with momentum P^{fin}. If the target was at $X = -d/2$, it reaches $X = d/2$ after a time Md/P^{fin}, while the probe wave packet requires a time $md\cos\theta^{in}/p^{in}$ to reach $X = d/2$ if it crosses $X = -d/2$ without scattering. If these times coincide, then the scattered target states in the superposition coincide, and their orthogonality is transferred to the probe. The condition for this transfer of orthogonality is $P^{fin} = Mp^{in}/m\cos\theta^{in}$. Since $P_*^{fin} = Mp^{in}/m\cos\theta^{in}$ is algebraically equivalent to $\partial P_*^{in}/\partial p^{in} = 0$, the condition that visibility not be suppressed implies transfer of orthogonality, here just as in the one-dimensional model [10].

A full experimental feasibility study will appear elsewhere [11]. Let us, however, apply our second example to a typical experimental setting in which only the final direction of the probe is measured. We have numerically integrated $p^{fin}\text{prob}(p_x^{fin}, p_y^{fin})$ with respect to p^{fin} along lines of constant θ^{fin}. In the numerical integration, we took $\varphi_L(X)$ and $\varphi_R(X)$ to have the form $e^{-(X\pm d/2)^2/2w^2}$ with $w = \lambda^{in}/5$, $d = 7\lambda^{in}$ and incident probe wavelength $\lambda^{in} = 0.5\mu$m. (Double wells with ground state of size 0.1 μm and separation 3.5 μm are achievable with magnetic traps.) While integrating over p^{fin} tends to average out some of the interference, Fig. 2 shows that the visibility is robust. The dependence of the scattering on the relative phase α is very clear. The insets show how visibility depends on the mass ratio M/m and on $\Delta p^{in}/p^{in}$. For $M/m > \cos^2\theta^{in}$ the visibility is suppressed, as we expect since $M/m > \cos^2\theta^{in}$ is incompatible with the condition $P_*^{fin} = Mp^{in}/m\cos\theta^{in}$.

In summary, we have shown how a distinctive new interferometry can yield the relative phase of superposed orthogonal location states of a free target.

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[7] For $m \ll M$ there is no “which path” information in the target recoil.

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[9] For the parameters of Fig. 2 there are no two values of p^{in} in the range $\Delta p^{in}/p^{in} < 0.3$ consistent with the same p_x^{fin}, p_y^{fin}, P^{fin}.

[10] Here we can impose the condition of perfect visibility by postselecting the probe state, i.e. by measuring also p^{fin}.

[11] In a longer calculation, we have generalized Eq. (18) to arbitrary initial probe states, including arbitrary uncertainties in momentum magnitude and direction.